國立高雄大學 100 學年度應用數學系
微積分競賽考題

學號：　　　　　　　　姓名：　　　　　　　　編號：

題號	Part I	Part II 1	2	3	4	5	總分
分數							

Part I: Fill in the blanks.

1. (15%) Determine whether the statement is True or False.

   _____ (a) The statement "Suppose $\epsilon = 1$ and let $\delta > 0$ be given. Choose $x = \max\{3, 4 - \frac{\delta}{2}\}$. Then $0 < |x - 4| < \delta$ and $|x - 2| \geq \epsilon$." can show that $\lim_{x \to 4} x \neq 2$.

   _____ (a) If $\lim_{x \to a} f(x) = f(a)$, then $f'(a)$ exists.

   _____ (a) $\int_0^3 \frac{1}{x-1} dx = \ln 2$.

   _____ (a) There is a power series whose interval of convergence is $[0, \infty)$.

   _____ (e) If f is continuous on $[0, 1]$, then $\int_0^1 f(x)dx = \int_0^1 f(1-x)dx$

2. (5%) Find the vertical and horizontal asymptotes of the graph of

$$f(x) = \frac{2x}{\sqrt{3x^2 - 1}}.$$

   Ans:
   Vertical asymptotes ______________________.
   Horizontal asymptotes______________________.

3. (10%) Evaluate the limit by first recognizing the sum as a Riemann sum for a function defined on $[0, 1]$:

$$\lim_{n \to \infty} \left(\frac{1}{\sqrt{n(n+1)}} + \frac{1}{\sqrt{n(n+2)}} + \cdots + \frac{1}{\sqrt{n(n+n)}} \right)$$

$$= \int_0^1 \_\_\_\_\_\_\_\_\_\_\_\_ \, dx = \_\_\_\_\_\_\_\_ .$$

4. (5%) $g(x) = \int_0^{\cos x} (1+\sin(t^2))dt$ and $f(x) = \int_0^{g(x)} \frac{1}{\sqrt{1+t^3}}dt$. Find $f'\left(\frac{\pi}{2}\right)$.

Ans:______________.

5. (5%) For $n = 0, 1, 2, \ldots$, that is, n is a nonnegative integer,

$$f_n(x) = \begin{cases} x^n \sin \frac{1}{x} & \text{if } x \neq 0 \\ 0 & \text{if } x = 0. \end{cases}$$

Find all n so that $f_n(x)$ is differentiable at $x = 0$.

Ans:______________.

6. (5%) Find the sum of the series $\sum_{n=0}^{\infty} \frac{(-1)^n \pi^{2n}}{6^{2n}(2n)!}$.

Ans:______________.

Part II: Calculations and/or Proofs (show all work).

1. (10%) Prove that $\lim_{x \to a} \sqrt{x} = \sqrt{a}$ if $a > 0$ using the δ, ϵ definition of limit.

2. (10%) Evaluate the integral $\int_0^{\pi/2} \frac{\sin 2x}{2+\cos x} dx$.

3. (10%) Find the volume common to two spheres, each with radius r, if the center of each sphere lies on the surface of the other sphere.

4. (10%) Find the radius of convergence and interval of convergence of the series

$$\sum_{n=0}^{\infty} \frac{(-3)^n x^n}{\sqrt{n+1}}.$$

5. (15%)

(a) Find the Maclaurin series of the function $f(x) = e^x$ using the definition and its radius of convergence.

(b) Prove that e^x is equal to the sum of its Maclaurin series.

(c) Evaluate $\int_0^1 e^{-x^2} dx$ correct to within an error of 0.01.