

國立高雄大學應用數學系 101 學年度 微積分競賽試題

學號：

姓名：

編號：

頁	1	2	3	4	總分
分數					

Part I: Fill in the blanks.

1. (15%) Determine whether the statement is True or False.

_____ (a) If $f(1) > 0$ and $f(3) < 0$, then there exists a number c between 1 and 3 such that $f(c) = 0$.

_____ (b) $\lim_{x \rightarrow \infty} \frac{x + \cos x}{x + 1}$ does not exist.

_____ (c) If $f'(c) = 0$, then f has a local maximum or minimum at c .

_____ (d) $f(x) = \int_0^x \sin(t^2 \arctan t) dt$ is differentiable on $(0, \infty)$.

_____ (e) If $\sum_{n=0}^{\infty} c_n 3^n$ is convergent, then $\sum_{n=0}^{\infty} c_n (-1)^n$ is convergent.

2. (5%) Let the symbol $[\]$ denote the greatest integer function. For what values of x is the function $f(x) = [x] + [-x]$ continuous?

Ans: _____.

3. (10%) Find $f^{(9)}(0)$ for $f(x) = x \cos(x^2)$.

Ans: _____.

4. (10%) Find the interval of convergence of the series $\sum_{k=1}^{\infty} \frac{1}{k 2^k} x^k$.

Ans: _____.

5. (10%) Find a and b given that the function $f(x) = \begin{cases} x^3 & x \leq 1 \\ ax + b, & x > 1 \end{cases}$ is differentiable at $x = 1$.

Ans: $(a, b) =$ _____.

Part II: Calculations and/or Proofs (show all work).

1. (20%) Suppose that $f(x)$ is continuous and increasing on $[-1, 2]$ with $f(x) > 0$. Let

$$F(x) = \int_0^x \left(t \int_1^t f(u) du \right) dt.$$

- (a) Find all critical points of $F(x)$ in $(-1, 2)$ and classify them (local minimum/maximum).
(b) Show that $F''(x)$ is increasing on $(0, 1)$ and there is a point of inflection of $F(x)$ on $(\frac{1}{2}, 1)$.

2. (10%) Calculate $\int \frac{x^4 + 2x^3 + 1}{(x - 1)(x^2 + 1)^2} dx$

3. (10%) Express the following limit as a definite integral of certain function and then evaluate the integral.

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \sum_{k=1}^n \frac{1}{\sqrt{n + \sqrt{nk}}}.$$

4. (10%) Find the area of the region that is outside $r = \cos 2\theta$ but inside $r = 1$, and sketch the polar curves.