

# 國立高雄大學九十九學年度應用數學系

## 微積分競賽考題

學號：

姓名：

編號：

頁	1	2	3	4	總分
分數					

### Part I: Fill in the Blanks.

1. (10 %) Determine whether the statement is True or False.

\_\_\_\_\_ (a) The product of two increasing functions is increasing.

\_\_\_\_\_ (b) If  $f'$  is continuous on  $[0, \infty)$  and  $\lim_{x \rightarrow \infty} f(x) = 0$ , then  $\int_0^{\infty} f'(x) dx = -f(0)$ .

\_\_\_\_\_ (c) If  $\sum a_n$  and  $\sum b_n$  both converge, then  $\sum a_n b_n$  converges.

\_\_\_\_\_ (d) It is possible to find a power series whose interval of convergence is  $[0, \infty)$ .

\_\_\_\_\_ (e) If  $f(x) = \sum_{n=0}^{\infty} a_n x^n$  converges for  $|x| < 2$ , then  $\int_0^1 f(x) dx = \sum_{n=0}^{\infty} \frac{a_n}{n+1}$ .

2. (5 %) Calculate  $\frac{d}{dx} \left( \int_{\tan x}^{x^2} \frac{1}{\sqrt{2+t^4}} dt \right)$ .

Ans: \_\_\_\_\_

3. (5 %) Find  $(f^{-1})'(-2)$  if  $f(x) = x\sqrt{3+x^2}$ .

Ans: \_\_\_\_\_

4. (5 %) Find the vertical and horizontal asymptotes for  $f(x) = \frac{x}{\sqrt{x^2-4}}$ .

Ans: \_\_\_\_\_

5. (5 %) Find  $f^{(9)}(0)$  for  $f(x) = x \cos(x^2)$ .

Ans: \_\_\_\_\_

6. (5 %) Determine whether  $\sum_{k=1}^{\infty} \left( \frac{n}{n+1} \right)^{n^2}$  converges or diverges. Identify the test used.

Ans: \_\_\_\_\_

7. (5 %) Find the sum of the series  $\sum_{n=0}^{\infty} \frac{(-1)^n}{3^n(2n+1)}$ .

Ans: \_\_\_\_\_

8. (5 %) Evaluate  $\lim_{x \rightarrow 0} \left( \frac{\sin x}{x} \right)^{1/x^2}$ .

Ans: \_\_\_\_\_

Part II: Calculations and/or Proofs. Show all work.

9. (5 %) Prove that if  $f$  is differentiable on  $(-\infty, \infty)$  and  $f'(x) < 1$  for all real numbers, then  $f$  has at most one fixed point. A fixed point of a function  $f$  is a real number  $c$  such that  $f(c) = c$ .

10. (10 %) Find the critical numbers, the relative extrema, and the points of inflection of

$$f(x) = \frac{x^3}{x^2 - 9}.$$

11. Find or evaluate the integral.

(a) (10 %)  $\int x \sin^{-1} x \, dx$

(b) (10 %)  $\int \frac{dx}{x^4 + 4}$

(c) (10 %)  $\int_0^{\pi/2} \frac{1}{3 - 2 \cos \theta} d\theta$

12. (10 %) Find the area of the surface generated by revolving  $y = \cos x$ ,  $x \in [0, \frac{\pi}{2}]$  about the  $x$ -axis.