

# NUK Math 徵答004 解答

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## 問題 004

Let  $a_1, a_2, \dots$  be positive numbers such that

$$\sum_{n=1}^{\infty} a_n < +\infty.$$

Prove that there are positive numbers  $c_1, c_2, \dots$  such that

$$\lim_{n \rightarrow \infty} c_n = \infty \quad \text{and} \quad \sum_{n=1}^{\infty} c_n a_n < +\infty.$$

解:

Since  $\sum_{n=1}^{\infty} a_n < +\infty$ , we can choose a sequence

$$1 < n_1 < n_2 < n_3 < \dots$$

in  $\mathbb{N}$  such that

$$\sum_{n=n_j}^{\infty} a_n < \frac{1}{j^3} \quad \forall j \in \mathbb{N}.$$

For any  $n \in \mathbb{N}$ , if  $1 \leq n < n_1$ , set  $c_n = 0$ ; if  $n_j \leq n < n_{j+1} - 1$ , set  $c_n = j$ . Then  $\lim_{n \rightarrow \infty} c_n = \infty$  and

$$\sum_{n=1}^{\infty} c_n a_n = \sum_{j=1}^{\infty} \sum_{n=n_j}^{n_{j+1}-1} c_n a_n \leq \sum_{j=1}^{\infty} j \sum_{n=n_j}^{\infty} a_n \leq \sum_{j=1}^{\infty} j \cdot \frac{1}{j^3} < \infty.$$

本題徵答無人完全答對。