NUK Math 徵答009 解答

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問題009

Show that very closed set in $\mathbb{R}$ is a countable intersection of open sets.

陳柏凱的解法

Since the statement "Every closed set in R is a countable intersection of open sets" is equivalent to "Every open set in $\mathbb{R}$ is a countable union of closed sets.", so we claim that the latter. First, $\mathbb{R}$ and $\emptyset$ are open and closed, so we have done it. Next, since every nonempty open subset in $\mathbb{R}$ could be written as a union of disjoint open intervals, moreover, every open interval is a class of (a,b), $(-\infty,b)$ and (a,∞), $a,b \in \mathbb{R}$. In particular,

$$(a,b) = \bigcup_{n=1}^{\infty}[a+1/n, b-1/n],\ (-\infty,b) = \bigcup_{n=1}^{\infty}(-\infty, b-1/n]$$

and

$$(a,\infty) = \bigcup_{n=1}^{\infty}[a+1/n,\infty).$$

Therefore, every open subset G in $\mathbb{R}$, there exists a countable collection F of disjoint closed intervals such that $G = \bigcup F$, then we complete the proof.

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