1. Let $x(t)$ and $y(t)$ are two functions defined on $[0, \infty)$, and L is a nonnegative constant. Assume $x(t)$ and $y(t)$ satisfy the following initial-valued problem of ODE,

$$\begin{aligned} \frac{dx}{dt} &= -x + \frac{L}{1+y} \\ \frac{dy}{dt} &= -y + 2x \\ x(0) &= x_0 \\ y(0) &= y_0 \end{aligned}$$

please prove that $x(t)$ and $y(t)$ must be nonnegative functions if $x_0 \geq 0$ and $y_0 \geq 0$.

Solution: Whenever $y = 0, dy/dt > 0$ because $x, L > 0$. Therefore y must become positive (or remain zero), but it cannot become negative. Similarly, $dx/dt > 0$ whenever $x = 0$, so x can never become negative. Thus, all trajectories starting in the first quadrant must remain there.