HEDGING RAINBOW OPTIONS IN DISCRETE TIME

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ABSTRACT

A quadratic hedging strategy based on minimizing the hedging costs at each pre-determined rebalancing time is used to hedge rainbow options. The corresponding hedging positions are simply obtained by solving a linear system, which is convenient for practical implementation. Quadratic hedging and the widely used delta hedging are compared to investigate their hedging performance for rainbow options in discrete time. By employing the Value-at-Risk (VaR) as the risk measure for comparing the hedging performance, simulation results indicate that the quadratic hedging performs better than the delta method in both static and dynamic scenarios.

Key words and phrases: change of numéraire, quadratic hedging, rainbow options.
JEL classification: C61, G11, G13.

1. Introduction

A rainbow option is a financial derivative whose payoff depends on two or more underlying assets. Margrabe (1978) first derived the pricing formula for a special type of rainbow options, namely the “Margrabe option” or “exchange option”, under the Black-Scholes framework. The holder of the exchange option has the right to exchange one asset for the other at maturity. Stulz (1982) further obtained the pricing formula for European call and put options on the maximum or minimum of two risky assets. Johnson (1987) extended the two-asset rainbow option to the multi-asset case and
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developed the closed-form pricing formula. Ouwehand and West (2006) employed the change of numéraire machinery to establish a tractable way for deriving the pricing formula of multi-asset rainbow options. In addition to the development of the pricing formula for rainbow options, the issuing bank further needs to form a hedging portfolio to cover its short position in practice.

A widely used hedging method for a contingent claim is delta hedging, which is formed by holding delta positions in the underlying assets. The delta values are defined as the first order partial derivatives of the value of the contingent claim with respect to each risky asset. By using the closed-form pricing formulae of rainbow options in the Black-Scholes framework, the delta values are obtained by straightforward computation (Ouwehand and West, 2006). Unfortunately, delta hedging is only instantaneous and continuous rebalancing is clearly impossible in practice. In the case of discrete rebalancing, the holding positions are then defined by certain optimum criteria, for example, by minimizing the hedging risks or maximizing investor’s utility. However, investigation of the hedging performance in discrete rebalancing between the delta hedging and other hedging strategies satisfying certain economic consideration for rainbow options is still lacking in the literature.

In the literature, several risk-minimizing criteria based on the discounted cumulative hedging cost were utilized to determine the hedging strategy, for instance, the total quadratic risk (QR) criterion (Duffie and Richardson, 1991; Schäl, 1994; Schweizer, 1995), the local QR criterion (Föllmer and Schweizer, 1991; Schäl, 1994), the local quadratic risk-adjusted (QRA) criterion (Elliott and Madan, 1998), and the local piecewise linear risk minimization criterion (Coleman et al., 2003). Schweizer (1995) proved the existence of a self-financing strategy under the total QR criterion with a mean-variance tradeoff condition. If the hedging strategy is allowed to be non-self-financing, then additional money can be invested into the hedging portfolio at each pre-determined rebalancing time. In this case, the optimal hedging scheme which minimizes the quadratic incremental discounted cost can be established by the local QR criterion. In this study, we use the local QR criterion to form a quadratic hedging portfolio for our investigation. The holding positions in the quadratic hedging portfolio are simply obtained by solving a linear system, which is convenient for practical imple-
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A comparison study of delta hedging and quadratic hedging is conducted in both static and dynamic hedging scenarios with consideration of transaction costs. The Value-at-Risk (VaR) is employed to assess the hedging performance of the two hedging strategies. Numerical results indicate that the quadratic hedging portfolio has smaller VaR than delta hedging in most cases of static hedging, especially when the hedging period or the expected return increases. Furthermore, when the number of risky assets increases, the performance of the quadratic hedging portfolio is also better than delta hedging by comparing the VaRs of the two hedging methods. In addition, the quadratic hedging is capable of avoiding more extreme loss than delta hedging in dynamic hedging cases even with consideration of transaction costs. Consequently, the quadratic hedging provides a tractable and promising way for hedging rainbow options in discrete rebalancing.

The rest of this paper is organized as follows. In Section 2, the derivation of the pricing formulae of rainbow options by the change of numéraire machinery is introduced. In Section 3, the delta and the quadratic hedging strategies are introduced. Simulation studies comparing the hedging performance of the two hedging strategies are presented in Section 4. Conclusions are in Section 5.

2. Rainbow option pricing

In this section, we briefly introduce the pricing scheme of rainbow options by the change of numéraire approach. The payoffs of several different types of rainbow options are listed in the following table, where $S_{i,T}$ denotes the price of the $i$th underlying asset at maturity $T$ and $K$ is the strike price. Obviously, the payoff of a rainbow option depends on the prices of multiple underlying assets and its pricing scheme has to deal with the dynamics of these risky assets.
<table>
<thead>
<tr>
<th>Type of Rainbow Options</th>
<th>Payoff</th>
</tr>
</thead>
<tbody>
<tr>
<td>Margrabe option</td>
<td>$\max(S_{1,T} - S_{2,T}, 0)$</td>
</tr>
<tr>
<td>Better-off option</td>
<td>$\max(S_{1,T}, S_{2,T}, \ldots, S_{n,T})$</td>
</tr>
<tr>
<td>Worse-off option</td>
<td>$\min(S_{1,T}, S_{2,T}, \ldots, S_{n,T})$</td>
</tr>
<tr>
<td>Binary maximum option</td>
<td>$1_{\max(S_{1,T}, S_{2,T}, \ldots, S_{n,T}) &gt; K)}$</td>
</tr>
<tr>
<td>Maximum call option</td>
<td>$\max(\max(S_{1,T}, S_{2,T}, \ldots, S_{n,T}) - K, 0)$</td>
</tr>
<tr>
<td>Minimum call option</td>
<td>$ \max(\min(S_{1,T}, S_{2,T}, \ldots, S_{n,T}) - K, 0)$</td>
</tr>
<tr>
<td>Spread option</td>
<td>$\max(\frac{S_{1,T} - S_{2,T} - \cdots - S_{n,T}}{n} - K, 0)$</td>
</tr>
<tr>
<td>Basket average option</td>
<td>$\max(\frac{S_{1,T} - S_{2,T} - \cdots - S_{n,T}}{n} - K, 0)$</td>
</tr>
<tr>
<td>Multi-strike option</td>
<td>$\max(\frac{S_{1,T} - K, S_{2,T} - K, \ldots, S_{n,T} - K}{n}, 0)$</td>
</tr>
</tbody>
</table>

In this study, we consider the following process for the risky assets under the physical (or dynamic) measure $\mathbb{P}$

$$dS_t/S_t = \mu dt + A dW_t$$

where $dS_t/S_t = (dS_{1,t}/S_{1,t}, \ldots, dS_{n,t}/S_{n,t})^T$, $S_{i,t}$ is the price of the $i$-th underlying asset at time $t$, $\mu = (\mu_1, \ldots, \mu_n)^T$, $\mu_i$ is the expected return of the $i$-th asset, $W_t = (W_{1,t}, \ldots, W_{n,t})^T$, $W_{i,t}$'s are independent Brownian motions, and

$$A = \begin{pmatrix}
    a_{11} & 0 & \cdots & 0 \\
    a_{21} & a_{22} & \ddots & \vdots \\
    \vdots & \ddots & \ddots & 0 \\
    a_{n1} & a_{n2} & \cdots & a_{nn}
\end{pmatrix}$$

which is defined by the Cholesky decomposition such that $AA^T = \Sigma = (\sigma_{ij})$, for $i, j = 1, \ldots, n$, where $\sigma_{ij}$ denotes the covariance of the $i$-th and $j$-th underlying assets. Moreover, let $\sigma_i$ be the instantaneous volatility of the $i$-th asset, $\rho_{ij}$ be the correlation of the $i$-th and $j$-th assets, and $a_i$ denote the $i$-th row of $A$. Then $\sigma_i$ and $\rho_{ij}$ can be represented by the entries of $A$ as follows:

$$\sigma_i^2 \equiv \sigma_{ii} = \sum_{j=1}^{n} a_{ij}^2 \quad \text{and} \quad \rho_{ij} \equiv \sigma_{ij}/\sigma_i \sigma_j = \frac{a_i a_j^T}{\|a_i\| \|a_j\|}$$

where $\| \cdot \|$ is the Euclidean norm. For example, if $n = 2$, the dynamics of the underlying assets $S_{1,t}$ and $S_{2,t}$ are

$$\begin{pmatrix} dS_{1,t} \\ dS_{2,t} \end{pmatrix} = \begin{pmatrix} \mu_1 dt \\ \mu_2 dt \end{pmatrix} + \begin{pmatrix} a_{11} & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} dW_{1,t} \\ dW_{2,t} \end{pmatrix}$$
and

\[ AA' = \Sigma = \begin{pmatrix} \sigma_1^2 & \rho_{12}\sigma_1\sigma_2 \\ \rho_{12}\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix} = \begin{pmatrix} a_{11} & 0 & 0 \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} a_{11} & a_{21} \\ 0 & a_{22} \end{pmatrix} = \begin{pmatrix} a_{11}^2 & a_{11}a_{21} \\ a_{11}a_{21} & a_{21}^2 + a_{22}^2 \end{pmatrix} \]

The risk-neutral counterpart of Model (1) is

\[ dS/S = rd\text{t} + Ad\tilde{W}_t \] (2)

where \( r = (r; \ldots; r)^T_{1 \times n} \), \( r \) is the risk-free interest rate and \( \tilde{W}_t = (\tilde{W}_{1,t}, \ldots, \tilde{W}_{n,t})^T \) is a vector of independent Brownian motion under the risk-neutral measure \( \mathbb{Q} \) with the same covariance structure as \( W \) under the physical measure \( \mathbb{P} \).

2.1 Change of numéraire method

Let \( V_t \) denote a no-arbitrage price of a European type derivative at time \( t \). Under a risk-neutral measure \( \mathbb{Q} \), the discounted price process \( \{V_t\} \) is a martingale, that is,

\[ V_t = e^{-r(T-t)}E^\mathbb{Q}_t (V_T), \quad 0 \leq t \leq T \] (3)

where \( E^\mathbb{Q}_t (\cdot) \) is the conditional expectation given the information up to time \( t \). For pricing a minimum call option of two risky assets, its closed-form solution can be obtained by computing (3) directly. However, it is difficult to extend the derivation for the 2 risky assets to the case of \( n > 2 \). Geman et al. (1995) solved this problem by using change of numéraire.

A numéraire is a price process \( \hat{A}_t \), which is almost surely strictly positive for each \( t \in [0, T] \). Let \( \hat{S}_t = S_t/\hat{A}_t \) denote the price process of \( S_t \) evaluated by a numéraire \( \hat{A}_t \). Then there exists an equivalent martingale measure \( \hat{\mathbb{Q}} \) with the numéraire \( \hat{A} \) such that

\[ E^\hat{\mathbb{Q}}_u (\hat{S}_t) = \hat{S}_u, \quad 0 \leq u \leq t \]

In addition, the price of the European derivative at time \( t \) can be obtained by

\[ V_t = \hat{V}_t \hat{A}_t \]

where \( \hat{V}_t = E^\hat{\mathbb{Q}}_t (\hat{V}_T) \).
For example, let $S_{j,t}$ be the numéraire, where $j \in \{1, \ldots, n\}$. Denote the ratio of the $i$th asset over the numéraire by $S_{i,t}^{(j)}$, that is,

$$S_{i,t}^{(j)} = \frac{S_{i,t}}{S_{j,t}} \quad \text{for} \quad i = 1, \ldots, n$$

By (2) and Ito’s lemma, $S_{i,t}^{(j)}$ satisfies

$$dS_{i,t}^{(j)} / S_{i,t}^{(j)} = (\|a_j\|^2 - a_i a_j^T) dt + (a_i - a_j) d\tilde{W}_i$$

under the risk-neutral measure $Q$. Furthermore, by using Girsanov theorem, there exists a $Q_j$ measure such that the process $\{S_{i,t}^{(j)}\}$ is a $Q_j$-martingale and

$$dS_{i,t}^{(j)} / S_{i,t}^{(j)} = (a_i - a_j) dW_t^{(j)}$$

where $W_t^{(j)}$ is an $n$-dimensional Brownian motion under measure $Q_j$ satisfying

$$(a_i - a_j) W_t^{(j)} = (\|a_j\|^2 - a_i a_j^T) t + (a_i - a_j) \tilde{W}_t$$

As a result, by (5), $\log S_{i,T}^{(j)}$ is normally distributed with mean $\log S_{i,0}^{(j)} - \frac{1}{2} \sigma_{i,j}^2 T$ and variance $\sigma_{i,j}^2 T$, where

$$\sigma_{i,j}^2 = \sigma_i^2 + \sigma_j^2 - 2\sigma_{ij}$$

In the following, we use three examples to illustrate the pricing scheme by using the change of numéraire method.

### 2.2 Margrabe option pricing

A Margrabe (also called exchange) option is evaluated in Margrabe (1978) to exchange one asset into another asset at the maturity $T$. The payoff is defined as

$$V_{M,T} = \max(S_{1,T} - S_{2,T}, 0)$$

Suppose that the second asset is selected to be the numéraire and the risk-free interest rate is zero. Then formula (7) can be explicitly written as

$$\frac{V_{M,T}}{S_{2,T}} = \frac{1}{S_{2,T}} \max(S_{1,T} - S_{2,T}, 0) = \max(S_{1,T} / S_{2,T} - 1, 0)$$

Consequently, the right-hand-side of (8) can be treated as a European call option with strike price 1 based on an underlying asset with price process $S_{1,t} / S_{2,t}$. By (5), the
pricing formula of the derivative with payoff $V_{M,T}/S_{2,T}$ defined in (8) can be obtained by the Black-Scholes framework. Therefore, the price of a Margrabe option is

$$V_{M,0} = S_{1,0}N(d_+) - S_{2,0}N(d_-)$$

where

$$d_\pm = \frac{\log(S_{1,0}/S_{2,0}) \pm \frac{\sigma_1^2 T}{2}}{\sigma_{1/2} \sqrt{T}}$$

$sigma_{1/2}$ is defined in (6) and $N(\cdot)$ is the distribution function of a standard normal random variable.

### 2.3 Better-off option pricing

To simplify the illustration, we use a better-off option based on three risky assets to describe the pricing scheme. The payoff for a better-off option based on three risky assets is defined as

$$V_{\text{max},T} = \max(S_{1,T}, S_{2,T}, S_{3,T})$$

If $S_{j,T}$ is the largest price, then choose $S_{j,T}$ to be the numéraire. By (4), we have

$$V_{\text{max},0} = S_{1,0}E^{Q_1}(I\{S_{1,T}^{(1)} < 1, S_{2,T}^{(1)} < 1\}) + S_{2,0}E^{Q_2}(I\{S_{1,T}^{(2)} < 1, S_{3,T}^{(2)} < 1\})
+ S_{3,0}E^{Q_3}(I\{S_{1,T}^{(3)} < 1, S_{2,T}^{(3)} < 1\})
= S_{1,0}N_2(-d_-^{1/1} - d_-^{3/1}, \rho_{2,3,1}) + S_{2,0}N_2(-d_-^{1/2} - d_-^{3/2}, \rho_{1,3,2})
+ S_{3,0}N_2(-d_-^{1/3} - d_-^{2/3}, \rho_{1,2,3})$$

where $I_{\{\cdot\}}$ is an indicator function,

$$d^{i/j}_\pm = -d^{i/j}_+ = \frac{\log(S_{i,0}/S_{j,0}) \pm \frac{\sigma_{i,j}^2 T}{2}}{\sigma_{i,j} \sqrt{T}}$$

with $\sigma_{i,j}$ defined in (6),

$$\rho_{i,j,k} = \frac{(a_i - a_k)(a_j - a_k)}{|a_i - a_k||a_j - a_k|}$$

is the correlation between $S_{i/k,T}$ and $S_{j/k,T}$ under the measure $Q_k$, $k = 1, 2, 3$, and $N_2(\cdot, \cdot, \rho)$ is the distribution function of a bivariate normal random variable with zero means, unit variances and correlation $\rho$. 
2.4 Rainbow call option pricing

A special case of the better-off option defined in (11) is

$$V_{\text{max},T} = \max(S_{1,T}, S_{2,T}, S_{3,T}) = \max(S_{1,T}, S_{2,T}, K)$$

where the third asset satisfies $S_{3,t} = Ke^{-r(T-t)}$. In this case, the third asset is independent of the other two assets and has zero volatility. Moreover, by equations (6), (13), and (14), we have $\sigma_3 = 0$, $\rho_{i,j,3} = \rho_{ij}$, $\sigma_{i/3} = \sigma_i = \sigma_{3/i}$ and $d_{3/2}^{\pm} = -d_{2/2}^{\pm}$. As a result, by (12), we have

$$V_{\text{max},0} = S_{1,0}N_2(-d_{1/1}^{2/1}, d_{2/2}^{1/1}, \rho_{2,3,1}) + S_{2,0}N_2(-d_{1/1}^{2/2}, d_{2/2}^{2/2}, \rho_{1,3,2}) + Ke^{-rT}N_2(-d_{1/1}^{1/1}, -d_{2/2}^{2/2}, \rho_{12})$$

(15)

Consequently, we can extend the result in (15) to rainbow call option pricing.

The payoff of a rainbow call on the max of two assets option is defined as

$$V_{c_{\text{max},T}} = \max(\max(S_{1,T}, S_{2,T}) - K, 0) = \max(S_{1,T}, S_{2,T}, K) - K$$

Hence, the no-arbitrage price of a rainbow call with payoff $V_{c_{\text{max},T}}$ is $V_{c_{\text{max},0}} = V_{\text{max},0} - Ke^{-rT}$, which is consistent with the formula of Stulz (1982), where $V_{\text{max},0}$ is given in (15). In general, the payoff of a rainbow call on the max of $n$ assets option is defined as

$$V_{c_{\text{max},T}} = \max(\max(S_{1,T}, \ldots, S_{n,T}) - K, 0) = \max_{1<i<n} (S_{i,T}, K) - K$$

and the corresponding no-arbitrage price is

$$V_{c_{\text{max},0}} = \sum_{k=1}^{n} S_{k,0}N_n(-d_{k/2}^{1/k}, d_{k/2}^{2/k}, \rho_{k,\cdot,k}) - Ke^{-rT}\{1 - N_n(-d_{1/1}^{1/1}, -d_{2/2}^{2/2}, \ldots, -d_{12}^{12}, \rho_{12}, \rho_{13}, \ldots)\},$$

(16)

where $N_n(\cdot)$ is the distribution function of an $n$-dimensional multivariate normal random variable with zero means and unit variances,

$$d_{i/2}^{\pm} = \frac{\log(S_{i,0}/K) + (r \pm \frac{1}{2}\sigma_i^2)T}{\sigma_i\sqrt{T}},$$

(17)
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\[ \mathbf{a}_{-k}^i = \begin{cases} (d_{-1}^{2/1}, \ldots, d_{-1}^{n/k}), & k = 1; \\ (d_{-1}^{1/k}, \ldots, d_{-1}^{(k-1)/k}, d_{-1}^{(k+1)/k}, \ldots, d_{-1}^{n/k}), & k = 2, \ldots, n-1; \\ (d_{-1}^{1/n}, \ldots, d_{-1}^{(n-1)/n}), & k = n; \end{cases} \tag{18} \]

with \( d_{-1}^{ij} \) defined in (13), and

\[ \mathbf{p}_{-k} = \begin{cases} (\rho_{2,3,1}, \ldots, \rho_{2,n+1,1}, \rho_{3,4,1}, \ldots, \rho_{3,n+1,1}, \ldots, \rho_{n,n+1,1}), & k = 1; \\ (\rho_{1,2,k}, \ldots, \rho_{1,k-1,k}, \rho_{1,k+1,k}, \ldots, \rho_{1,n+1,k}, \ldots), & k = 2, \ldots, n-1; \\ (\rho_{1,2,n}, \ldots, \rho_{1,n-1,n}, \rho_{1,n+1,n}, \rho_{2,3,n}, \ldots, \rho_{n-1,n+1,n}), & k = n; \end{cases} \tag{19} \]

with \( \rho_{i,j,k} \) defined in (14). Note that equation (16) is consistent with equation (1) in Johnson (1987). One of the advantages of using the change of numéraire method to derive (16) is that it is more trackable than the approach in Johnson (1987). Moreover, the pricing formula of a rainbow call option on the minimum of \( n \) risky assets can be obtained by similar arguments (Ouwehand and West, 2006). In addition, the rainbow put options on the maximum or minimum of the risky assets can be evaluated by the put-call parities for rainbow options: \( V_{p_{\text{max}},0} = Ke^{-rT} - V_{c_{\text{max}},0}(K = 0) + V_{c_{\text{max}},0} \) and \( V_{p_{\text{min}},0} = Ke^{-rT} - V_{c_{\text{min}},0}(K = 0) + V_{c_{\text{min}},0} \).

3. Hedging Strategies of Rainbow Options

In the financial literature, delta hedging is a commonly used hedging strategy. Unfortunately, delta hedging performs worse as the hedging period increases (Huang and Guo, 2012). In the following, we employ a quadratic hedging strategy based on minimizing the hedging risks in each rebalancing period and investigate the hedging performance of the quadratic and delta hedging methods for rainbow options in both static and dynamic scenarios.

3.1 Static hedging

Consider establishing a static hedging strategy for a seller of a rainbow option with initial hedging capital \( F_0 \). The hedging strategy consists of the risky underlying assets
and a money market account. That is,

\[ F_0 = C_0 + \sum_{i=1}^{n} w_{i,0} S_{i,0} \]  

(20)

where \( C_0 \) is the value in the money market account and \( w_i \)'s are the positions of the underlying assets \( S_i, i = 1, \ldots, n \), respectively. Holding the hedging portfolio till the maturity \( T \), the value of the portfolio, denoted by \( F_T \), becomes

\[ F_T = C_0 e^{rT} + \sum_{i=1}^{n} w_{i,0} S_{i,T} \]

In this study, we consider the case that the initial hedging capital is equal to the no-arbitrage price of the rainbow option. That is, \( F_0 = V_0 \), where the value of \( V_0 \) can be obtained by the corresponding pricing formula of the hedging target in Section 2. Therefore, the objective is to compute the hedging positions \( w_{i,0} \)'s.

3.1.1 Delta hedging for rainbow options

The delta hedging positions of a Margrabe option and a rainbow option on maximum call are given for illustration in this section. By definition, the delta values for a Margrabe option in (20) are \( w_{1,0} = \Delta_{1,0} \equiv \partial V_{M,0}/\partial S_{1,0} = N(d_+) \) and \( w_{2,0} = \Delta_{2,0} \equiv \partial V_{M,0}/\partial S_{1,0} = -N(d_-) \), where \( V_{M,0} \) is defined in (9), and \( d_+ \) and \( d_- \) are defined in (10).

Similarly, by the pricing formula of the rainbow call on the max of \( n \) assets option in (16), the corresponding delta hedging positions in the hedging portfolio (20) are

\[ (w_{1,0}, \ldots, w_{n,0})^T = (\Delta_{1,0}, \ldots, \Delta_{n,0})^T = \left( \frac{\partial V_{c_{\max},0}}{\partial S_{1,0}}, \ldots, \frac{\partial V_{c_{\max},0}}{\partial S_{n,0}} \right)^T \]  

(21)

where \( \Delta_{k,0} = N_n(-d_{\,-1}^{i,k}, d_{\,+1}^{i,k}, \rho_{i,j,k}) \) for \( k = 1, \ldots, n \), in which \( d_{\,\pm}^{i,k} \), \( d_{\,\pm}^{i,k} \) and \( \rho_{i,j,k} \) are defined in (17), (18) and (19), respectively.

3.1.2 Quadratic hedging for rainbow options

The hedging positions of the hedging portfolio (20) are obtained by the following criterion:

\[ \min_{w_{1,0}, \ldots, w_{n,0}} E(V_T - F_T)^2 \quad \text{subject to} \quad F_0 = V_0 \]  

(22)
where $V_T$ denotes the payoff. In particular, under the Black-Scholes model, the optimal $w_{i,0}$’s are obtained by the following proposition.

**Proposition 3.1.** Consider the static hedging strategy defined in (20) under Model (1). The optimal hedging positions satisfying the criterion (22) are

$$ \theta = A^{-1}B $$

where $\theta = (w_{1,0}, \ldots, w_{n,0}, C_0)^T$, $B = (E(V_T S_{1,T}), \ldots, E(V_T S_{n,T}), V_0)^T$ and

$$ A = \begin{pmatrix}
E(S^2_{1,T}) & E(S_{1,T}S_{2,T}) & \cdots & E(S_{1,T}S_{n,T}) & e^{rT}E(S_{1,T}) \\
E(S_{1,T}S_{2,T}) & E(S^2_{2,T}) & \cdots & E(S_{2,T}S_{n,T}) & e^{rT}E(S_{2,T}) \\
\vdots & \vdots & \ddots & \vdots & \vdots \\
E(S_{1,T}S_{n,T}) & E(S_{2,T}) & \cdots & E(S^2_{n,T}) & e^{rT}E(S_{n,T}) \\
S_{1,0} & S_{2,0} & \cdots & S_{n,0} & 1
\end{pmatrix} $$

in which

$$ E(S_{i,T}) = S_{i,0}e^{\mu_i T}, \quad \text{if } 1 \leq i \leq n $$
$$ E(S^2_{i,T}) = S_{i,0}^2e^{(\mu_i + \sigma_i^2)T}, \quad \text{if } 1 \leq i \leq n $$
$$ E(S_{i,T}S_{j,T}) = S_{i,0}S_{j,0}e^{(\mu_i + \mu_j + \rho_{ij}\sigma_i\sigma_j)T}, \quad \text{if } i \neq j, \quad 1 \leq i \leq n, 1 \leq j \leq n $$

and $E(V_T S_{1,T}), \ldots, E(V_T S_{n,T})$ can be obtained by using Monte Carlo simulation method.

Note that the result in Proposition 3.1 can be simply obtained by using the Lagrange multiplier method.

### 3.2 Dynamic hedging

In dynamic hedging, the holding positions of the hedging portfolio are recalculated at each rebalancing time, denoted by $t_k$, $k = 1, \ldots, N$, satisfying $0 = t_0 < t_1 < \ldots < t_N = T$. At $t_0$, the hedging portfolio is set as (20), that is,

$$ F_0 = C_0 + w_{1,0}S_{1,0} + \cdots + w_{n,0}S_{n,0} $$

and assume $F_0 = V_0$. Holding this portfolio till $t_1$, the value of the hedging portfolio becomes

$$ F_{t_1} = C_0e^{r(t_1-t_0)} + w_{1,t_0}S_{1,t_1} + \cdots + w_{n,t_0}S_{n,t_1} $$ (23)
prior to rebalancing. After rebalancing at time $t_1$, the hedging portfolio is set as

$$F_{t_1} = C_{t_1} + w_{1,t_1}S_{1,t_1} + \cdots + w_{n,t_1}S_{n,t_1}$$  \hspace{1cm} (24)

and assume $F_{t_1} = V_{t_1}$, where $V_{t_1}$ is the no-arbitrage price of the rainbow option when the underlying asset prices are $S_{1,t_1}, \ldots, S_{n,t_1}$. Denote the loss of the hedging portfolio at $t_1$ by

$$D_{t_1} = V_{t_1} - F_{t_1}$$ \hspace{1cm} (25)

By similar arguments used in (23)-(25) at each time $t_k$, $k = 2, \ldots, N$, we obtain the total loss of the hedging portfolio, that is,

$$G = \sum_{k=1}^{N} D_{t_k} e^{r(T-t_k)}$$

To investigate the impact of transaction costs on the delta and the quadratic hedging methods, assume that proportional transaction costs are incurred when trading the risky securities in a hedging portfolio prior to maturity (Boyle and Vorst, 1992; Liu, 2004). Denote the transaction costs at time $t_k$, $k = 1, \ldots, N - 1$, by

$$H_{t_k} = \xi(|w_{1,t_k} - w_{1,t_{k-1}}|S_{1,t_k} + \cdots + |w_{n,t_k} - w_{n,t_{k-1}}|S_{n,t_k})$$

where $\xi$ is the transaction costs rate measured as a fraction of the amount traded. Then the loss of the hedging portfolio with transaction costs at time $t_k$ is

$$D_{t_k}^* = D_{t_k} + H_{t_k}, \quad k = 1, \ldots, N - 1, \quad \text{and} \quad D_{t_N}^* = D_{t_N}.$$

Hence, the total loss of the hedging portfolio with transaction costs is

$$G^* = \sum_{k=1}^{N} D_{t_k}^* e^{r(T-t_k)}$$ \hspace{1cm} (26)

The main task is to calculate the hedging positions $(w_{1,t_k}, \ldots, w_{n,t_k})$ at each rebalancing time $t_k$, $k = 0, 1, \ldots, N - 1$. In dynamic delta hedging, $(w_{1,t_k}, \ldots, w_{n,t_k})$ are computed by the first order partial derivatives of $V_{t_k}$ with respect to $(S_{1,t_k}, \ldots, S_{n,t_k})$, respectively, which is similar to (21) in the static hedging, where $V_{t_k}$ denotes the no-arbitrage price of the rainbow option at time $t_k$. As for the quadratic hedging method, $(w_{1,t_k}, \ldots, w_{n,t_k})$ are established by the criterion

$$\min_{w_{1,t_k}, \ldots, w_{n,t_k}} \mathbb{E}(V_{t_{k+1}} - F_{t_{k+1}})^2 \quad \text{subject to} \quad F_{t_k} = V_{t_k}$$
where \( F_{t_k^+} = C_{t_k} + w_{1,t_k} S_{k,t_k} + \cdots + w_{n,t_k} S_{n,t_k} \), for \( k = 0, \ldots, N - 1 \). Then the solution of \((w_{1,t_k}, \ldots, w_{n,t_k})\) can be obtained by Proposition 3.1 after replacing the time indexes 0 and \( T \) by \( t_k \) and \( t_{k+1} \), respectively.

4. Simulation Study

In this section, we perform several simulation studies to compare the static and dynamic hedging performance of the quadratic and delta hedging strategies. The VaR at the 0.95 level, defined by \( \text{VaR}_{0.95} = \sup \{ z \in R; P(\text{Loss} \leq z) \leq 0.95 \} \), is employed to assess the hedging performance.

4.1 Simulation study of static hedging

First, we investigate the static hedging performance of the two hedging strategies for rainbow call options on the maximum of two underlying assets. Herein, we use \( F_{t}^{\Delta} \) to denote the value of the delta hedging portfolio and \( F_{t}^{Q} \) for the quadratic hedging method at time \( t \). The simulation procedure is as follows:

(i) Set up the delta and the quadratic hedging portfolios at time 0 by (20), where the hedging positions of delta hedging, denoted by \((w_{1,0}^{\Delta}, w_{2,0}^{\Delta})\), are given in (21) and the hedging positions of quadratic hedging, denoted by \((w_{1,0}^{Q}, w_{2,0}^{Q})\), are obtained by Proposition 3.1.

(ii) Generate the stock prices \((S_{1,T}^{(j)}, S_{2,T}^{(j)})\) by Model (1), where the super-index \( j \) stands for the \( j \)-th random path, \( j = 1, \ldots, m \).

(iii) Calculate

\[
D_0 = \frac{1}{m} \sum_{j=1}^{m} D_0^{(j)} \quad \text{and} \quad D_1 = \frac{1}{m} \sum_{j=1}^{m} D_1^{(j)}
\]

where \( D_0^{(j)} = (V_T^{(j)} - F_T^{\Delta^{(j)}})/V_0 \) and \( D_1^{(j)} = (V_T^{(j)} - F_T^{Q^{(j)}})/V_0 \), for \( j = 1, \ldots, m \), denote the average unit loss of delta and quadratic hedging, respectively, in which \( V_T^{(j)} = \max(\max(S_{1,T}^{(j)}, S_{2,T}^{(j)}) - K, 0) \), \( F_T^{\Delta^{(j)}} = C_0 e^{rT} + w_{1,0}^{\Delta} S_{1,T}^{(j)} + w_{2,0}^{\Delta} S_{2,T}^{(j)} \) and \( F_T^{Q^{(j)}} = C_0 e^{rT} + w_{1,0}^{Q} S_{1,T}^{(j)} + w_{2,0}^{Q} S_{2,T}^{(j)} \).
(iv) Compute the ratio of

$$RVaR_{0.95} = \frac{VaR_{0.95}^0}{VaR_{0.95}^1}$$

where $VaR_{0.95}^0$ and $VaR_{0.95}^1$ are the empirical $\alpha$th-quantile of $\{D_0^{(j)}\}_{j=1}^m$ and $\{D_1^{(j)}\}_{j=1}^m$, respectively. If $RVaR_{0.95} > 1$, then the quadratic method performs better than delta hedging.

Table 1 gives the static hedging performance of the two strategies for rainbow call options on the maximum of two risky assets with different asset prices and maturities based on 50,000 random paths. In Table 1, $\bar{D}_1$ is smaller than $\bar{D}_0$ in all cases and the values of $RVaR_{0.95}$ are greater than 1 in most cases. This phenomenon indicates that quadratic hedging strategy has smaller average loss than delta hedging as well as having smaller VaR at the 95% confidence level than delta hedging. Therefore, the quadratic hedging method involves smaller hedging risks than delta hedging in this static scenario.

Table 1  Static hedging performance of the delta hedging and the quadratic hedging strategies for rainbow options on max call with 50,000 random paths, where $D_0$ and $D_1$ are defined in (27), $r = 0.05$, $\mu_1 = \mu_2 = 0.2$, $\sigma_1 = \sigma_2 = 0.30$ and $\rho_{12} = 0.50$.

<table>
<thead>
<tr>
<th>T</th>
<th>$S_{1,0}/K, S_{2,0}/K$</th>
<th>$\bar{D}_0$</th>
<th>$\bar{D}_1$</th>
<th>$RVaR_{0.95}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>(0.95, 1)</td>
<td>(1, 1)</td>
<td>0.0067</td>
<td>-0.0070</td>
<td>1.0022</td>
</tr>
<tr>
<td>10</td>
<td>(1, 1)</td>
<td>0.0053</td>
<td>-0.0050</td>
<td>1.0064</td>
</tr>
<tr>
<td></td>
<td>(1.05, 1)</td>
<td>0.0004</td>
<td>-0.0024</td>
<td>0.9845</td>
</tr>
<tr>
<td>(0.95, 1)</td>
<td>(1, 1)</td>
<td>0.0015</td>
<td>-0.0044</td>
<td>1.0138</td>
</tr>
<tr>
<td>20</td>
<td>(1, 1)</td>
<td>0.0019</td>
<td>-0.0006</td>
<td>0.9971</td>
</tr>
<tr>
<td></td>
<td>(1.05, 1)</td>
<td>0.0034</td>
<td>0.0032</td>
<td>0.9954</td>
</tr>
<tr>
<td>(0.95, 1)</td>
<td>(1, 1)</td>
<td>0.0039</td>
<td>-0.0031</td>
<td>1.0201</td>
</tr>
<tr>
<td>30</td>
<td>(1, 1)</td>
<td>0.0034</td>
<td>0.0032</td>
<td>0.9954</td>
</tr>
<tr>
<td></td>
<td>(1.05, 1)</td>
<td>0.0039</td>
<td>-0.0031</td>
<td>1.0201</td>
</tr>
</tbody>
</table>

Moreover, to investigate the effect of model parameters, $\mu$, $\sigma$ and $\rho$, on the hedging performance, we generate $m_1$ random pairs of the initial asset prices $(S_{1,0}, S_{2,0})$, and compute the corresponding sample mean

$$\overline{Y} = \frac{1}{m_1} \sum_{j=1}^{m_1} Y_i \quad (28)$$
where $Y_i = I_{\{RVaR_{0.95,i}>1\}}$, $i = 1, \ldots, m_1$, in which $RVaR_{0.95,i}$ denotes the ratio of the VaRs of delta and the quadratic hedging, based on the $i$-th pair of $(S_{1,0}, S_{2,0})$ and 50,000 random paths. If the quadratic hedging method has smaller VaR than delta hedging with a fixed pair of initial stock prices, then the value of the indicator function $Y_i$ is one. If this phenomenon holds for most of the initial stock prices, then $\bar{Y}$ should be greater than 0.5. Tables 2-4 show the simulation results of $\bar{Y}$ with two risky assets and $m_1 = 1,000$, where the standard deviation of $\bar{Y}$ is obtained by the bootstrap method.

As shown in Table 1.2 of Tsay (2010), the means of daily log returns of selected indexes and stocks are between 0.023% and 0.095%, and the associated standard deviations are between 0.816% and 2.905%. That is, the means of log returns are between 0.06 and 0.24, and the standard deviations are between 0.13 and 0.46 annually if we set 1 year equals 252 trading days. In addition, since we consider short term ($T \leq 30$ days) hedging horizon in the following, most liquid options with maturity less than and equal to 30 days have the ratio of $S_0/K$ between 0.95 and 1.05. Table 2 shows the results with $\sigma_1 = 0.2, 0.3, 0.4, 0.5$; Table 3 presents the results with $\mu_1 = 0.1, 0.15, 0.2, 0.25$; Table 4 shows the results with $\rho_{12} = -0.9, -0.6, -0.3, 0, 0.3, 0.6, 0.9$, where other parameter settings are the same as in Table 1 and the initial asset prices are generated independently from a $\text{Unif}(0.95K;1.05K)$ distribution. In these tables, most of the $\bar{Y}$ are significantly greater than 0.5, which indicates that the quadratic hedging portfolio provides better protection than the commonly used delta hedging, especially when the hedging period or the expected return increase.

Considering a higher dimensional case, Table 5 presents the values of $\bar{Y}$ and the corresponding standard deviations with dimension $n = 2, 3, 5, 7$, where $r = 0.05$, $K = 100$ and $T = 10, 20, 30$ days. In particular, to remove the effects of the initial stock prices, expected returns and volatility structure on the comparison study, the values of these initial settings are generated randomly. More precisely, the initial stock prices $S_{i,0}, i = 1, \ldots, n$, are generated independently from a $\text{Unif}(0.95K;1.05K)$ distribution, $\mu_i$’s are generated independently from a $\text{Unif}(0.1, 0.25)$ distribution, $\sigma_i$’s are generated independently from a $\text{Unif}(0.2, 0.5)$ distribution and $\rho_{ij}$’s are generated independently from a $\text{Unif}(-0.9, 0.9)$ distribution in each of the $m_1 = 1,000$ replication. In Table 5, $\bar{Y}$’s are significantly greater than 0.5 in all cases, especially when the number of risky
assets or length of hedging period increase.

Table 2  Simulation results of \( \bar{Y} \) defined in (28) with \( m_1 = 1,000, \mu_1 = \mu_2 = 0.2, \sigma_1 = 0.3, \rho_{12} = 0.5 \) and \( r = 0.05 \), where standard deviations are obtained by the bootstrap method.

<table>
<thead>
<tr>
<th>( T )</th>
<th>( Y )</th>
<th>( \sigma_1 )</th>
<th>( \sigma_2 )</th>
<th>( \sigma_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.6740</td>
<td>0.5770</td>
<td>0.5630</td>
<td>0.5220</td>
</tr>
<tr>
<td>std.</td>
<td>0.0142</td>
<td>0.0153</td>
<td>0.0153</td>
<td>0.0153</td>
</tr>
<tr>
<td>20</td>
<td>0.8810</td>
<td>0.8840</td>
<td>0.8580</td>
<td>0.8170</td>
</tr>
<tr>
<td>std.</td>
<td>0.0101</td>
<td>0.0103</td>
<td>0.0104</td>
<td>0.0128</td>
</tr>
<tr>
<td>30</td>
<td>0.9670</td>
<td>0.9660</td>
<td>0.9470</td>
<td>0.9300</td>
</tr>
<tr>
<td>std.</td>
<td>0.0055</td>
<td>0.0055</td>
<td>0.0072</td>
<td>0.0080</td>
</tr>
</tbody>
</table>

Cases where \( \bar{Y} > 0.5 \) significantly are denoted by boldface.

Table 3  Simulation results of \( \bar{Y} \) defined in (28) with \( m_1 = 1,000, \mu_2 = 0.2, \sigma_1 = \sigma_2 = 0.3, \rho_{12} = 0.5, \) and \( r = 0.05 \), where the standard deviations are obtained by the bootstrap method.

<table>
<thead>
<tr>
<th>( T )</th>
<th>( Y )</th>
<th>( \mu_1 )</th>
<th>( \mu_2 )</th>
<th>( \mu_3 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.5140</td>
<td>0.5060</td>
<td>0.6030</td>
<td>0.7040</td>
</tr>
<tr>
<td>std.</td>
<td>0.0158</td>
<td>0.0161</td>
<td>0.0155</td>
<td>0.0137</td>
</tr>
<tr>
<td>20</td>
<td>0.8230</td>
<td>0.8200</td>
<td>0.8730</td>
<td>0.9410</td>
</tr>
<tr>
<td>std.</td>
<td>0.0119</td>
<td>0.0120</td>
<td>0.0104</td>
<td>0.0073</td>
</tr>
<tr>
<td>30</td>
<td>0.9350</td>
<td>0.9480</td>
<td>0.9640</td>
<td>0.9890</td>
</tr>
<tr>
<td>std.</td>
<td>0.0074</td>
<td>0.0070</td>
<td>0.0057</td>
<td>0.0032</td>
</tr>
</tbody>
</table>

Cases where \( \bar{Y} > 0.5 \) significantly are denoted by boldface.

4.2 Simulation study of dynamic hedging

The simulation procedure of dynamic hedging strategy is as follows:

(i) Set up the delta and the quadratic hedging portfolios at time 0 by the method introduced in Section 3.2.

(ii) Generate the stock prices \( (S_{1,t_k}^{(j)}, \ldots, S_{n,t_k}^{(j)}) \) by Model (1), where \( j = 1, \ldots, m \) and \( k = \{1, \ldots, N\} \).
HEDGING RAINBOW OPTIONS

Table 4  Simulation results of $\overline{Y}$ defined in (28) with $m_1 = 1,000$, $\mu_1 = \mu_2 = 0.2$, $\sigma_1 = \sigma_2 = 0.3$, and $r = 0.05$, where the standard deviations are obtained by the bootstrap method.

<table>
<thead>
<tr>
<th>$\rho_{12}$</th>
<th>-0.9</th>
<th>-0.6</th>
<th>-0.3</th>
<th>0</th>
<th>0.3</th>
<th>0.6</th>
<th>0.9</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=10$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\overline{Y}$</td>
<td>0.7420</td>
<td>0.7450</td>
<td>0.7330</td>
<td>0.7160</td>
<td>0.6370</td>
<td>0.5880</td>
<td>0.7100</td>
</tr>
<tr>
<td>std.</td>
<td>0.0138</td>
<td>0.0137</td>
<td>0.0142</td>
<td>0.0142</td>
<td>0.0157</td>
<td>0.0156</td>
<td>0.0150</td>
</tr>
<tr>
<td>$T=20$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\overline{Y}$</td>
<td>0.8770</td>
<td>0.8990</td>
<td>0.8870</td>
<td>0.8950</td>
<td>0.8680</td>
<td>0.8740</td>
<td>0.9430</td>
</tr>
<tr>
<td>std.</td>
<td>0.0105</td>
<td>0.0098</td>
<td>0.0101</td>
<td>0.0098</td>
<td>0.0107</td>
<td>0.0104</td>
<td>0.0074</td>
</tr>
<tr>
<td>$T=30$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\overline{Y}$</td>
<td>0.9410</td>
<td>0.9570</td>
<td>0.9580</td>
<td>0.9610</td>
<td>0.9710</td>
<td>0.9760</td>
<td>0.9890</td>
</tr>
<tr>
<td>std.</td>
<td>0.0076</td>
<td>0.0065</td>
<td>0.0063</td>
<td>0.0061</td>
<td>0.0052</td>
<td>0.0048</td>
<td>0.0033</td>
</tr>
</tbody>
</table>

Cases where $\overline{Y} > 0.5$ significantly are denoted by boldface.

Table 5  Values of $\overline{Y}$ and the corresponding standard deviations with $n = 2, 3, 5, 7$, where $K = 100$, $T = 10, 20, 30$ days, $m_1 = 1,000$ and the initial settings are generated randomly from uniform distributions, as illustrated in Section 4.1.

<table>
<thead>
<tr>
<th>$n$</th>
<th>2</th>
<th>3</th>
<th>5</th>
<th>7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T=10$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\overline{Y}$</td>
<td>0.6190</td>
<td>0.6740</td>
<td>0.7360</td>
<td>0.7780</td>
</tr>
<tr>
<td>std.</td>
<td>0.0155</td>
<td>0.0144</td>
<td>0.0141</td>
<td>0.0126</td>
</tr>
<tr>
<td>$T=20$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\overline{Y}$</td>
<td>0.8330</td>
<td>0.8310</td>
<td>0.8500</td>
<td>0.8950</td>
</tr>
<tr>
<td>std.</td>
<td>0.0124</td>
<td>0.0120</td>
<td>0.0111</td>
<td>0.0094</td>
</tr>
<tr>
<td>$T=30$</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\overline{Y}$</td>
<td>0.9130</td>
<td>0.8920</td>
<td>0.9050</td>
<td>0.9220</td>
</tr>
<tr>
<td>std.</td>
<td>0.0090</td>
<td>0.0065</td>
<td>0.0091</td>
<td>0.0088</td>
</tr>
</tbody>
</table>

Cases where $\overline{Y} > 0.5$ significantly are denoted by boldface.

(iii) Calculate $G^{(j)} = \sum_{k=1}^{N} G_{t_k}^{(j)} e^{r(T-t_k)}$, where $G_{t_k}^{(j)} = V_{t_k}^{(j)} - F_{t_k}^{(j)}$, and $F_{t_k}^{(j)}$ is defined in (24), for both hedging strategies. If transaction costs are considered, then compute the total gain of the hedging portfolio with transaction costs by (26) with $\xi = 0.01$, that is, $G^{*^{(j)}} = \sum_{k=1}^{N} G_{t_k}^{*^{(j)}} e^{r(T-t_k)}$, where $G_{t_k}^{*^{(j)}} = G_{t_k}^{(j)} + H_{t_k}^{(j)}$ and

$$H_{t_k}^{(j)} = 0.01(\sum_{m=1}^{n} |w_{1,t_k} - w_{1,t_k-1}|S_{1,t_k}^{(j)} + \cdots + |w_{n,t_k} - w_{n,t_k-1}|S_{n,t_k}^{(j)})$$

(iv) Compute the ratio of RVaR$_{0.95} = \text{VaR}^{0}_{0.95}/\text{VaR}^{1}_{0.95}$, where VaR$_{0}$ and VaR$_{1}$ are the empirical $\alpha$-th quantile of $\{G_{0}^{(j)}\}_{j=1}^{m}$ and $\{G_{1}^{(j)}\}_{j=1}^{m}$, respectively. Similarly, if transaction costs are considered, then compute the ratio of RVaR$_{0.95}^* = \text{VaR}^{0*}_{0.95}/\text{VaR}^{1*}_{0.95}$, where RVaR$^*$ is the empirical $\alpha$-th quantile of $\{G_{0}^{*^{(j)}}\}_{j=1}^{m}$ and $\{G_{1}^{*^{(j)}}\}_{j=1}^{m}$.
where \( \text{VaR}_0^{\alpha} \) and \( \text{VaR}_1^{\alpha} \) are the empirical \( \alpha \)-th quantile of \( \{ G_0^{\ast(j)} \} \) and \( \{ G_1^{\ast(j)} \} \), respectively.

Simulation results for the dynamic strategy are shown in Table 6 based on 10,000 random paths, where \( r = 0.05 \), \( \mu_1 = \mu_2 = 0.1, 0.2 \), \( \sigma_1 = 0.20, 0.35, 0.50 \), \( \sigma_2 = 0.30 \) and \( \rho_{12} = 0.50 \). In Table 6, the values of RVaR\(_{0.95}^{\ast} \) and RVaR\(_{0.95}^{\ast} \) are all greater than 1, which indicates that the quadratic method has smaller VaR than delta hedging even when transaction costs are considered. Similar to the phenomenon shown in the static scenario (see Table 3), the superiority of quadratic hedging over delta hedging increases in terms of the ratio of their VaRs when the expected returns, \( \mu_1 \) and \( \mu_2 \), of the underlying assets increase. Note that the higher the expected return is, the more opportunity of the rainbow option having positive payoff at maturity is. As a result, the issuing bank of the rainbow option demands more to hedge it in this situation. The numerical results in Table 6 indicate that quadratic hedging provides a promising way to avoid extreme loss when using dynamic hedging strategy for rainbow options in discrete time.

Table 6  Dynamic hedging performance of the delta hedging and the quadratic hedging strategies for rainbow options on max call with \( r = 0.05 \), \( \sigma_2 = 0.30 \), \( \rho_{12} = 0.5 \) and 10,000 random paths.

<table>
<thead>
<tr>
<th>((S_{1.0}/K, S_{2.0}/K))</th>
<th>(\sigma_1)</th>
<th>(\mu_1 = \mu_2 = 0.1)</th>
<th>(\mu_1 = \mu_2 = 0.2)</th>
</tr>
</thead>
<tbody>
<tr>
<td>((0.95, 1))</td>
<td>0.2</td>
<td>1.6323</td>
<td>1.5947</td>
</tr>
<tr>
<td>((0.95, 1))</td>
<td>0.35</td>
<td>1.9753</td>
<td>1.9497</td>
</tr>
<tr>
<td>((0.95, 1))</td>
<td>0.5</td>
<td>1.7893</td>
<td>1.7646</td>
</tr>
<tr>
<td>((1, 1))</td>
<td>0.2</td>
<td>1.7612</td>
<td>1.7329</td>
</tr>
<tr>
<td>((1, 1))</td>
<td>0.35</td>
<td>1.5995</td>
<td>1.5806</td>
</tr>
<tr>
<td>((1, 1))</td>
<td>0.5</td>
<td>1.4877</td>
<td>1.4632</td>
</tr>
<tr>
<td>((1.05, 1))</td>
<td>0.2</td>
<td>1.4495</td>
<td>1.4279</td>
</tr>
<tr>
<td>((1.05, 1))</td>
<td>0.35</td>
<td>1.9180</td>
<td>1.8967</td>
</tr>
<tr>
<td>((1.05, 1))</td>
<td>0.5</td>
<td>1.8119</td>
<td>1.7843</td>
</tr>
</tbody>
</table>
5. Conclusion

This study investigates the hedging performance of delta hedging and quadratic hedging strategies for rainbow options in discrete rebalancing. Simulation results indicate that quadratic hedging performs better than delta hedging in the static hedging case, especially when the time period of rebalancing, the expected return of the underlying assets or the number of risky assets increase. In addition, quadratic hedging is capable of avoiding more extreme loss than delta hedging in dynamic hedging cases even with consideration of transaction costs. As a result, quadratic hedging strategy provides a handy and promising hedging scheme for multiple underlying products in discrete rebalancing.

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References


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